Potential metrics for use in evaluating configurations for the Pterodactyl UAV. The final metric will most likely be a combination of the ideas presented here.

**Determinant**

For this section, A is defined as the matrix that converts thrusts to torques and one force value. So, in the terminology of linear algebra, the 4x4 matrix is a linear transformation from thrust space to control space. It is well known that the determinant of the linear transformation matrix is the ratio of the volume of the new vector space to the volume of the old vector space. Another way of think of this is as a measure of thruster effectiveness.  
  
 There are two problems with this metric: the first is that it combines torque and force without any kind of scaling so it relies on the assumption that the different scales are reasonably similar. Fortunately when the body force along the z axis is the force component considered, the values are approximately or exactly -1 so the force component doesn’t change things too much.  
 The second problem is that as currently used it can’t account for thrust in the body x and y directions.

**Moment of Inertia**

The reasoning behind this method is apparent: the moments of inertia determine how the vehicle responds to applied torques. Aircraft with high moments of inertia are less agile but more stable whereas aircraft with lower moments of inertia are capable of much faster orientation changes but have the problem that they are highly susceptible to external disturbances such as wind gusts.

Additionally, the moments of inertia matrix can be compared to the A matrix described in the previous section to ensure that the moment of inertia about a given axis is roughly proportional to the available torque about that axis so that the stability/agility about all axes are roughly equal.  
  
 The final advantage of this metric is that it is useful not just for determining geometric constraints but also for more refined design decisions such as placement of batteries and control electronics.

The primary disadvantage of this metric is that its validity depends greatly on the accuracy of the model used to generate it. As the design process iterates and a higher fidelity model is available, this metric will be relied upon more and more.

**Condition Number**

For this section, A is a 6x4 matrix with the top half converting thrusts to torques and the bottom half converting the same thrusts to body forces. The problem for controlling 6 DOF with 4 actuators can be thought of as an overconstrained linear system of equations. The best (least sum of squared error) solution to this system can be found using the Moore-Penrose psuedoinverse. In this case the condition number of A is a measure of how good of an expected solution can be found. A few experiments with this method gave mixed results. It might not be feasible or it might just need proper gain tuning to work.  
  
**Null Space Analysis**

The steady state condition of the vehicle in hover involves (essentially) no torque and just force in the inertial frame –z direction. By calculating the orthonormal basis of the null space of the top half of the A matrix defined in the previous section, we get a unit vector that produces forces without torque. Multiplying the bottom half of A by this vector gives the unit vector in body frame describing the orientation of the resultant force vector. From this vector the rotational displacement required to hover without torque is simple to calculate. Obviously we want the configuration that requires the smallest displacement.  
  
 Other than neglecting BEM torques, the only way I have found so far to get the body forces with no torque in the body –z direction is to make the hinge line perpendicular to the leading edge.